Energy conditions bounds and supernovae data

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The energy conditions play an important role in the description of some important properties of the Universe, including the current accelerating expansion phase and the possible recent phase of super-acceleration. In a recent work we have provided a detailed study of the energy conditions for the recent past by deriving bounds from energy conditions and by making the confrontation of the bounds with supernovae data. Here, we extend and update these results in two different ways. First, by carrying out a new statistical analysis for $q(z)$ estimates needed for the confrontation between the bounds and supernovae data. Second, by providing a new picture of the energy conditions fulfillment and violation in the light of the recently compiled *Union* set of 307 type Ia supernovae and by using two different statistical approaches.

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I. INTRODUCTION

In the study of the classical energy conditions [1] in cosmological context, an important viewpoint is the confrontation of their predictions with the observational data. Since the pioneering papers by Visser [2] a number of articles have been published concerning this confrontation by using model-independent energy-conditions *integrated* bounds on the cosmological observables such as the distance modulus and lookback time [3] – [10] (see also the related Refs. [11]). Energy conditions constraints on modified gravity models, such as the so-called $f(R)-$gravity, have also been investigated in Ref. [12] and more recently in Ref. [13].

In a recent work [10], we have shown that the fulfillment (or the violation) of these *integrated* bounds at a given redshift z is not sufficient (nor necessary) to ensure the fulfillment (or the violation) of the energy conditions at z. This amount to saying that the local confrontation between the prediction of the *integrated* bounds and observational data is not sufficient to draw conclusions on the fulfillment (or violation) of the energy conditions at z. This crucial drawback in the confrontation between *integrated* bounds and cosmological data has been overcome in Ref. [10], where new *non-integrated* bounds have been derived, and confronted with type Ia supernovae (SNe Ia) data of the *gold* [14] and *combined* [15] samples.

In this letter, to proceed further with the investigation of the interrelation between energy conditions on scales relevant for cosmology and observational data, we extend and update the results of Ref. [10] in two different ways. First, carry out a new statistical analysis for $q(z)$ estimates necessary for the confrontation between the *non-integrated* bounds and supernovae data. Second, we give a new picture of the energy conditions fulfillment and violation for recent past ($z \leq 1$) by using the recently compiled *Union* sample [16] with 307 type Ia supernovae along with the new as well as the previous [10] statistical tools.

II. NON-INTEGRATED BOUNDS FROM THE ENERGY CONDITIONS

In order to use the energy conditions on cosmological scale, we consider the standard cosmological approach in which the Universe is modelled by a $4-$dimensional space-time manifold endowed with a locally homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (1)$$

where the spatial curvature $k = 0, 1$ or -1 and $a(t)$ is the scale factor. We additionally assume that the large scale structure of the Universe is determined by the gravitational interaction, and hence can be described by the General Relativity theory. These assumptions restrict the energy-momentum tensor to that of a perfect fluid of density ρ and pressure p, i.e., $T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu}$. In this context, the energy conditions take the following forms: [1]

$$\begin{aligned}
&\text{NEC}: && \rho + p \geq 0\,, && \\
&\text{WEC}: && \rho \geq 0 && \text{and} \quad \rho + p \geq 0\,, \\
&\text{SEC}: && \rho + 3p \geq 0 && \text{and} \quad \rho + p \geq 0\,, \\
&\text{DEC}: && \rho \geq 0 && \text{and} \quad -\rho \leq p \leq \rho\,,
\end{aligned} \quad (2)$$

where NEC, WEC, SEC and DEC correspond, respectively, to the null, weak, strong, and dominant energy

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conditions, and the density ρ and pressure p of the cosmological fluid are given by

$$\rho \;=\; \frac{3}{8\pi G}\left[\,\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\,\right]\,, \tag{3}$$

$$p \;=\; -\frac{1}{8\pi G}\left[\,2\,\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\,\right]\,, \tag{4}$$

where overdots denote the derivative with respect to the time t and G is the Newton's gravitational constant.

The *non-integrated* bounds from energy conditions [10] can then be obtained in terms of the deceleration parameter $q(z) = -\ddot{a}/aH^2$, the normalized Hubble function $E(z) = H(z)/H_0$, and the curvature density parameter $\Omega_{k0} = -k/(a_0H_0)^2$, simply by substituting Eqs. (3) into Eqs. (2) to give

$$\textbf{NEC} \;\;\Leftrightarrow\;\; q(z) - \Omega_{k0}\frac{(1+z)^2}{E^2(z)} \;\geq -1\;, \tag{5}$$

$$\textbf{WEC} \;\;\Leftrightarrow\;\; \frac{E^2(z)}{(1+z)^2} \;\geq \Omega_{k0}\;, \tag{6}$$

$$\textbf{SEC} \;\;\Leftrightarrow\;\; q(z) \;\geq 0\;, \tag{7}$$

$$\textbf{DEC} \;\;\Leftrightarrow\;\; q(z) + 2\,\Omega_{k0}\,\frac{(1+z)^2}{E^2(z)} \;\leq 2\;, \tag{8}$$

where $z = (a_0/a) - 1$ is the redshift, $H(z) = \dot{a}/a$, and the subscript 0 stands for present-day quantities. Here and in what follows we have used the notation of Ref. [10], in which **NEC**, **WEC**, **SEC** and **DEC** correspond, respectively, to $\rho + p \geq 0$, $\rho \geq 0$, $\rho + 3p \geq 0$ and $\rho - p \geq 0$.

We note that for a given spatial curvature Ω_{k0}, **NEC** [Eq.(5)] and **DEC** [Eq. (8)] provide, respectively, a lower and an upper bound on the $E(z)$–$q(z)$ plane for any fixed redshift $z_\star$. The **WEC** bound [Eq.(6)] only restricts the normalized Hubble function for a fixed value of Ω_{k0}, while the **SEC** bound [Eq.(7)] does not depend on the value of the spatial curvature. Thus, for any given value of Ω_{k0}, having estimates of $q(z_\star)$ and $E(z_\star)$ for different redshifts $z_\star$, one can test the fulfillment or violation of the energy conditions at each $z_\star$.

In this work, we focus on the FLRW flat ($\Omega_{k0} = 0$) universe, in which the **NEC** and **DEC** bounds reduce, respectively, to $q(z) \geq -1$ and $q(z) \leq 2$. Now, the $q(z_\star)$ and $E(z_\star)$ estimates are obtained by using a SNe Ia dataset, through a model-independent approach which consists in approximating the deceleration parameter $q(z)$ function in terms of the following linear piecewise continuous function (linear spline):

$$q(z) = q_l + q_l'\,\Delta z_l\;, \quad z \in (z_l, z_{l+1})\;, \tag{9}$$

where the subscript l means that the quantity is taken at z_l , $\Delta z_l \equiv (z - z_l)$, and the prime denotes the derivative with respect to z. The supernovae observations provide the redshifts and distance modulus

$$\mu(z) \;=\; 5\,\log_{10}\left[\frac{c}{H_0\,1\mathrm{Mpc}}\,\frac{(1+z)}{\sqrt{|\,\Omega_{k0}\,|}} \times\,\mathrm{S}_k\!\left(\sqrt{|\,\Omega_{k0}\,|}\int_0^z\frac{\mathrm{d}z'}{E(z')}\right)\right] + 25\;, \tag{10}$$

where $\mathrm{S}_k(x) = \sin(x)$, x, $\sinh(x)$ for $k = 1, 0, -1$, respectively. Then, by using the following relation between $q(z)$ and $E(z)$:

$$E(z) = \exp\int_0^z \frac{1+q(z)}{1+z}\,\mathrm{d}z\;, \tag{11}$$

along with Eq. (10), we fitted the parameters of the $q(z)$, as given by (9), by using the SNe Ia redshift–distance modulus data from the so-called *Union* sample as compiled by Kowalski *et al* [16].

III. RESULTS AND DISCUSSIONS

We have used two different statistical approaches to confront the energy conditions bounds with observational data. In the first approach, which holds only for the flat case[1], we have computed the $q(z_\star)$ estimates by marginalizing over $E(z_\star)$ and the other parameters (q_l''s) of the $q(z)$ function, with $1\sigma - 3\sigma$ confidence levels (C.L.). In the second procedure (which was used in Ref. [10]), we have calculated the $1\sigma - 3\sigma$ confidence regions on the $E(z_\star) - q(z_\star)$ plane, and used the upper and lower limits of $q(z)$ to have the $1\sigma - 3\sigma$ C.L. of $q(z)$ for all $z \leq 1$ (recent past).

To obtain a global picture of the violation and fulfillment of the energy conditions in the recent past by using the first statistical method, we have calculated the $q(z_\star)$ estimates at 200 equally spaced redshifts in the interval $(0, 1]$, and our result are depicted in Fig. 1a which shows the **NEC**, **SEC**, and **DEC** bounds along with the best-fit values and the 1σ, 2σ and 3σ limits of $q(z)$ in the $q(z) - z$ plane. We note that **WEC** bound is fulfilled identically ($E^2(z) \geq 0$) in the flat case.

When an observational confrontation is needed for a non-flat FLRW case ($\Omega_{k0} \neq 0$) the second statistical procedure has to be employed, because in this case the **NEC** [Eq. (5)] and **DEC** [Eq. (8)] bounds depend on the estimates of $E(z_\star)$. In this way, $q(z_\star)$ estimates cannot be obtained by marginalizing over $E(z_\star)$, and one has to calculate the confidence regions on the plane $E(z_\star) - q(z_\star)$ (second approach). In this work, however, we have used this approach also for the flat case (which we have focussed on) in order to make a comparison of the observational SNe confrontations obtained by using both statistical procedures, and also with the results of Ref. [10].

[1] Note that in this case the **NEC**, **SEC**, and **DEC** bounds are independent of $E(z)$.

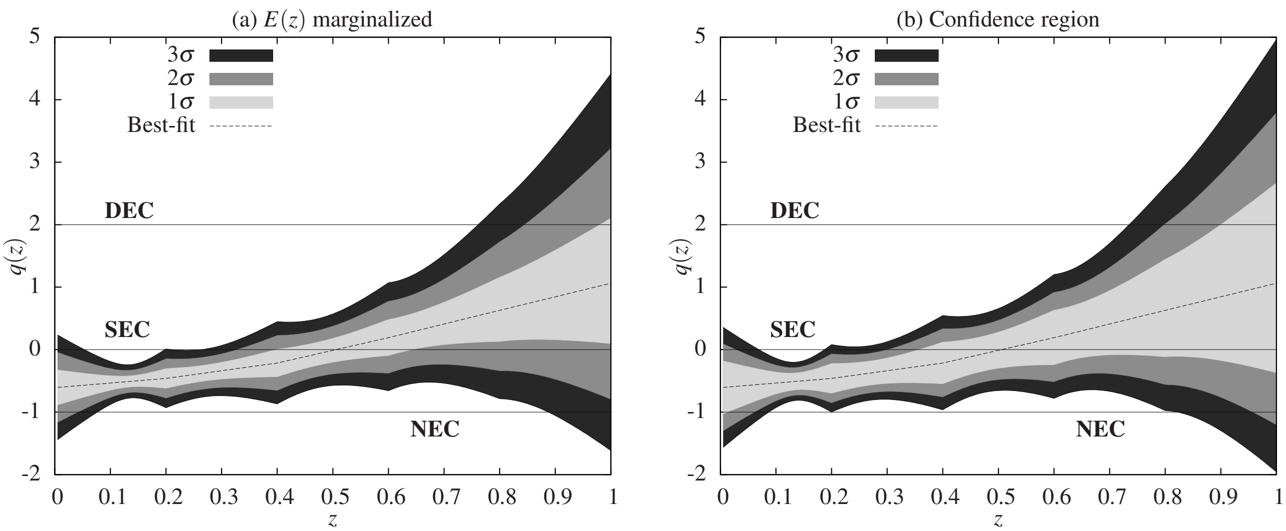

FIG. 1: The best-fit, the upper and lower 1σ, 2σ and 3σ limits of $q(z)$ estimates, marginalizing over $E(z)$ [panel (a)], and from the confidence regions on the $E(z) - q(z)$ plane [panel (b)], for 200 equally spaced redshifts. The **NEC** and **SEC** lower bounds, and also the **DEC** upper bound for the flat case are shown. This figure shows that the **SEC** is violated with 1σ confidence level until $z \simeq 0.4$ [panel (a)] and $z \simeq 0.36$ [panel (b)]. It also shows that, for both statistical methods employed, the **DEC** and **NEC** is violated for high redshifts within 3σ confidence level, while the **NEC** is violated for $z \lesssim 0.075$ [panel (a)] and $z \lesssim 0.085$ [panel (b)].

Figure 1(b) contains the result of our analysis obtained by using the second procedure. Besides the **NEC**, **SEC**, and **DEC** bounds, it shows the best-fit values, the upper and lower $1\sigma - 3\sigma$ limits of $q(z)$ from the confidence regions on the $E(z_\star) - q(z_\star)$ plane calculated for each of previously used equally spaced 200 redshifts in the interval $(0, 1]$.

The two panels in Fig. 1 show the violation of the **SEC** with more than 3σ in the redshift intervals $(0.05, \simeq 0.2)$ [panel (a)] and $(0.07, \simeq 0.19)$ [panel (b)], where the highest evidence of **SEC** violation is at $z = 0.133$ with 5.69σ [panel (a)] and 5.34σ [panel (b)]. Unlike the result from the confidence regions approach displayed in panel (b), which indicates the breakdown of **SEC** within $1\sigma - 3\sigma$ in the whole redshift interval, we note that from panel (a) one has that, within 1σ, the **SEC** is fulfilled for $z \gtrsim 0.64$ and is violated for $z \lesssim 0.4$. This indicates that, within 1σ C.L., the Universe crosses over from a decelerated to an accelerated expansion phase for a redshift within the interval $(\simeq 0.4, \simeq 0.64)$.

Regarding the **NEC**, Fig. 1 indicates its breakdown within 3σ for low redshift intervals, i.e., $(0, 0.075)$ [panel (a)] and $(0, 0.085)$ [panel (b)]. For higher values of redshift, **NEC** is violated with 3σ for $(z \gtrsim 0.89)$ [see panel (a)] and for $z \gtrsim 0.825$ [cf. panel (b)].

Concerning the **DEC**, we note that its fulfillment takes place in most of the redshift interval for both statistical analyses [see panel (a) and panel (b)], but it is violated within 3σ for $z \gtrsim 0.765$ [panel (a)] and $z \gtrsim 0.74$ [panel (b)], which are intervals where the error bars of our estimates grow significantly.

The comparison of the results obtained through the confidence regions approach by using the *Union* set of 307 SNe Ia with those of Ref. [10] calculated through the same statistical procedure but by employing the 182 and the 192 supernovae of the *gold* and *combined* samples, shows that the errors bars of $q(z_\star)$ from the Union sample analysis are smaller for redshifts lying in $(0, \simeq 0.7)$. The Union sample results reinforce the indication of the **SEC** violation and **NEC** fulfillment at low redshift pointed out recently in Ref. [10].

Here, similarly to the analyses of Ref. [10], we have found that the results of the analyses for the best fit, the upper and lower 1σ values of $\Omega_{k0} = -0.0046^{+0.0066}_{-0.0067}$ as given by five-year WMAP [17], are essentially the same of the flat case, with differences much smaller than the associated errors. We note that for the upper 1σ limit of $\Omega_{k0} = 0.002$, the **WEC** bound $[E^2(z_\star) \geq 0.002(1 + z_\star)^2]$ is fulfilled with 3σ confidence level in the redshift interval $(0, 1]$, while for the Ω_{k0} interval $(-0.0113, 0)$ the **WEC** is identically satisfied.

IV. CONCLUDING REMARKS

In a previous work [10] we provided a picture of the violation and fulfillment of the energy conditions in the recent past by deriving *non-integrated* bounds from energy conditions in terms of the deceleration parameter and the normalized Hubble function in the context of FLRW cosmology, and made the confrontation of the bounds with SNe Ia data through estimates of $q(z)$ from $1\sigma - 3\sigma$ confidence regions on the plane $E(z) - q(z)$ calculated with *gold* [14] and *combined* [15] samples.

Here, we have extended and updated the confrontation between the *non-integrated* bounds and supernovae data. First, by using the fact that, in the flat case, the **NEC**, **SEC**, and **DEC** bounds do not dependent on $E(z)$, we have carried out a new statistical analysis in which the $q(z)$ estimates are obtained by marginalizing over the parameters $E(z)$ along with the q_l''s of $q(z)$ [see Eq. (9)]. Second, we have updated the previous work [10] by providing a new picture [see Fig. 1(a) and [Fig. 1(b)] of the energy conditions fulfillment and violation from the recently compiled Union set of 307 SNe Ia along with two different statistical tools.

On general grounds, our analyses indicate a possible recent phase of super-acceleration in which the **NEC** is violated within 3σ confidence level for $z \lesssim 0.075$ [Fig. 1(a)] and $z \lesssim 0.085$ [Fig. 1(b)], and that the **DEC** is fulfilled with 3σ in the redshift interval $(0, 0.765)$ [Fig. 1(a)] and $(0, 0.74)$ [Fig. 1(b)]. Regarding the **SEC**, our analyses show that, for both statistical approaches employed, the best-fit curve of $q(z)$ crosses the **SEC**–bound curve at $z \simeq 0.51$, and that **SEC** is violated with 3σ within small low redshift intervals [Fig. 1(a) and [Fig. 1(b)].

Finally, an interesting fact that comes out of our **SEC** analysis with 1σ C.L., obtained by using the recent SNe Ia *Union* set, is that for the new $q(z)$ estimate [calculated by marginalizing over $E(z)$] the deceleration to acceleration transition expansion phase of the universe took place in the redshift interval $(\simeq 0.4, \simeq 0.64\,)$.

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